Optimisation of Building and Road Network Densities in Terms of Variation in Building Lot Sizes and shapesl

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Abstract: Urban form at the district scale is defined as a unique combination of the following basic spatial objects: road networks, building lot patterns and building configurations. In particular, a building lot represents one of the most important basic spatial objects of urban form. In recent theoretical and empirical work from downtown districts in Tokyo, it was found that: (1) building lot sizes and frontages approximately follow a log-normal distribution; and (2) the parameters can be primarily estimated by building density (the number of buildings per unit area) and road network density (total length of road networks per unit area). The rationale for the building and road network densities presented here will be discussed by considering the variation in building lot sizes and frontages through a stochastic approach. Allowing for the smaller building lot size than minimum building lot size regulation as well as the narrower building lot frontage than minimum building lot frontage regulation (these minimum criteria being determined by Japanese building codes), the maximum building density and optimal road network density will be obtained. These findings are expected to provide urban planners with a theoretical basis to discuss the validity of these two density values (based on the deterministic approach that assumes uniform urban form) in terms of evaluating the policy effect, such as minimum building lot size and frontage regulation.

1. Introduction

Urban form is defined as a unique combination of the following basic spatial objects: road networks, building lot patterns and building configurations. The combination of road networks and the series of urban blocks (consisting of lots and buildings) that are enclosed within these road networks constitute a *district* – a collection of urban blocks and large-scale non-built areas – in a two-dimensional plane (Berghauser Pont and Haupt, 2009; Kropf, 1996; Marshall, 2009; Oliveira, 2016). In particular, a building lot represents one of the most important basic spatial objects of urban form and tends to directly connect road networks (Barreira-González and Barros, 2016; Bitner *et al.*, 2009; Legras and Cavailhès, 2016; Marshall, 2009; Moudon, 1994; Oliveira, 2016; Pont and Haupt, 2009; Shayesteh and Steadman, 2015; Suen and Tang, 2002; Vialard and Carpenter, 2012). A similar phenomenon in urban space is termed *urban syntax* (Marshall, 2009). This mutual dependence of road networks and building lots is important because the open space of road networks enables light to access buildings

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and influences privacy depending on road width and building lot sizes. Wide road widths can compensate for small urban blocks and building lots and vice versa. Thus, the holism of road networks, building lots and buildings at a district scale should be central to a new definition of density (Berghauser Pont and Haupt, 2009; Dovey and Pafka, 2019).

Although a single density index is insufficient to an understanding of urban form, a method has been proposed to overcome this difficulty in classifying urban form (Berghauser Pont and Haupt, 2007, 2009): the combination of several density indices, called *Spacematrix*, which is defined as a three-dimensional diagram whose axes are the building coverage ratio (BCR), floor area ratio (FAR) and road network density (total length of road networks per unit area) of a district. Given that BCR can be estimated from building density (the number of buildings per unit area) (Koshizuka and Kotoh, 1989), *Spacematrix* can be redefined as a three-dimensional diagram whose axes are building density, road network density and FAR. Nevertheless, it has been found that: (1) building lot sizes, frontages and depths tend to follow a log-normal distribution irrespective of building density, road network density and patterns (whether regular grid or not); and (2) its mean and variance can be primarily estimated from building density and road network density (Usui 2018, 2019; Usui and Asami, 2019). Therefore, the building and road network densities of a district can also characterise the variation in building lot sizes, frontages and depths on the sub-space of *Spacematrix* (Usui, 2019). Conversely, it is expected that the variation in urban form as a set of building lots can be controlled by the combination of building density and road network density at a district scale. These findings will help provide urban planners with a theoretical framework, not only for a typology of urban form but also to discuss the rationale for present building and road network densities based on their relationship with variation in urban form.

The substantive motivation of this research is to provide a theoretical framework to discuss the rationale for present building and road network densities by considering variation in building lot sizes and frontages. According to Japanese building codes, the minimum building lot frontage criterion of two metres must be legally satisfied. However, for a better residential environment it is insufficient , and so a second minimum building lot frontage level might be proposed. Furthermore, minimum building lot size criterion can be legally determined in order to prevent the residential environment from deteriorating. Given these regulations, the upper ratio of the number of building lots that fail to meet these regulations is pre-determined as a criterion. If the ratio of the number of building lots is below a given criterion, the combination of current building and road network densities are deemed appropriate. However, the relationship between the ratio and combination of building density and road network density has yet to be investigated. Thus, urban planners tend to discuss the validity of present and future density values without examining variation in building lot sizes and frontages. Therefore, the following research question is expected to provide a theoretical foundation in determining the criteria for building and road network densities: *How can we optimise building density and road network density at a district scale subject to the ratio of number of building lots that do not meet the minimum building lot size and frontage regulations?*

The answer may contribute to delineating a feasible domain on the two-dimensional sub-space of *Spacematrix*. In setting the minimum building lot size regulation in a district (e.g., more than 100 m^2), we can statistically determine the maximum building density such that the probability that building lot sizes are smaller than the regulation value is below a given criterion. Furthermore, building density criteria have been used to define districts as densely built-up areas. In particular, a high building density, equivalent to a small average building lot size, is considered a major factor in the spread of fires and poor quality of sanitation, ventilation and privacy in Japan, rendering the residential environment worse as a whole. However, it is difficult to understand the diversity of building lot sizes and frontages in a densely built-up area (more than 60 buildings per hectare). The criteria for building density and road network density should be determined based not on a deterministic approach that assumes that all building lots are uniform in size and shape, but by considering the ratio of building lots smaller than the minimum building lot size criterion (typically 100 m^2), which may need to be enlarged. Nevertheless, the rationale behind this deterministic approach has not been discussed in the literature, necessitating its reconsideration through a comparison of deterministic and stochastic approaches. Given that urban planners may encounter data limitations regarding building lot shapes, we can offer a theoretical framework to: (1) optimise building and road network densities by considering the variation in building lot sizes and frontages based on a stochastic approach; and (2) discuss the rationale for current building and road network densities compared with optimal versions.

This paper is organised as follows. Based on the probability density function of building lot sizes and frontages, the second section will derive two inequalities regarding building density and explain how building and road network densities can be optimised. In the third section, the rationale for present building and road network densities in districts in the Tokyo metropolitan region will be discussed based on comparison with optimal criteria. In the final section, we provide concluding remarks and directions for future work.

2. Optimisation of building density and road network density

In this section, we explain how to establish maximum building density through consideration of the variation in building lot sizes and shapes. First, the probability density function of building lot sizes and frontages will be explained. Second, two inequalities regarding building density will be derived. Third, building density and road network density will be optimised. Hereafter, we consider a district of area *A*, length of perimeter *L*, total road length Λ, and number of building lots *n*. As regards districts in the Tokyo metropolitan region, the average *n* is approximately six hundred. According to the Building Standard Law of Japan (Japanese building codes), in order to appropriately regulate building size and shape by BCR and FAR in its lot, a building lot basically has no more than one building, termed the rule of one building lot for one building. Thus, by assuming that the rule of one building lot for one building holds, the number of buildings is equal to *n*.

Hereafter, the size and frontage of building lots are denoted by *s* and *F*, respectively. According to Usui and Asami (2019), the distribution of building lot sizes is log-normal based on Gibrat's law:

$$g(s) = \frac{1}{s\sqrt{2\pi\sigma_s^2}} \exp\left[-\frac{(\ln s - \mu_s)^2}{2\sigma_s^2}\right], \;\; s > 0, \tag{1}$$

where and are the mean and variance of ln *s*, respectively. These two parameters can be estimated from the following equations:

$$\mu_S = \ln \frac{1}{\sqrt{\eta^2+1}} \cdot \frac{\kappa(w,\lambda)}{\rho_{\mathrm{G}}}, \tag{2}$$

$$\sigma_S^2 = \ln(\eta^2 + 1), \tag{3}$$

where $\rho_G = n/A$ and η are building density and the coefficient of variation in building lot sizes, respectively. In addition, denotes the ratio of total building lot sizes to *A* as follows:

$$\kappa(w, \lambda) \approx 1 - w\lambda, \tag{4}$$

,

where λ denotes the road network density, Λ/*A*, and *w* mean road width. Furthermore, according to Usui (2018), building lots frontages follow a log-normal distribution:

$$h(F) = \frac{1}{F\sqrt{2\pi\sigma_F^2}} \exp\left[-\frac{(\ln F - \mu_F)^2}{2\sigma_F^2}\right], \quad F > 0, \tag{5}$$

,

where μ_F and σ_F^2 are the mean and variance of ln *F*, respectively. These two parameters can be estimated from the following equations:

$$\mu_F = \ln \frac{1}{\sqrt{\eta_F^2+1}} \cdot \frac{\lambda(2-w\lambda)}{\alpha\rho_{\mathrm{G}}}, \tag{6}$$

$$\sigma_F^2 = \ln(\eta_F^2 + 1), \tag{7}$$

where α denotes the ratio of the number of frontages to *n* and η_F denotes the coefficient of variation in building lot frontages. Based on the sensitivity analysis of *g(s)* and *h(F)* regarding *w* and α, changes in the values of these parameters scarcely affect the shapes of *g(s)* and *h(F)*. Moreover, η and η_F tend to be in the neighbourhood of 1.0 and 0.6, respectively (Usui 2018; Usui and Asami 2019). Thus, once these values are given as constant irrespective of district, and can be estimated primarily from ρ_G and λ.

Based on Equations (1) to (7), two inequalities regarding building density will be derived. Hereafter, the minimum building lot size is denoted by s_{mim}. By using Equations (1) to (4), the probability that a building lot size is less than s_{mim} can be formulated as:

,

$$G(s \le s_{\min}) = \int_0^{s_{\min}} g(s)ds \tag{8}$$

$$= \Phi\left(z_{\min} = \frac{\ln s_{\min} - (\ln(1-w\lambda) - \ln\rho_{\mathrm{G}} - \ln\sqrt{\eta^2+1})}{\sqrt{\ln(\eta^2+1)}}\right) \equiv \theta \le \theta_c,$$

where $\theta_c \in (0, 1)$ denotes the allowance for smaller building lot size than s_{mim}, and Φ(*z*) denotes the cumulative distribution function of the standard normal distribution. Since Φ(*z*) is the monotonically increasing function of *z*, we can define *z*min and *z*θc as the standardisation of ln s_{mim} and ln s such that $\Phi(z\theta c) = \theta c$, respectively. Therefore, Equation (8) is equivalent to the following:

$$\frac{\ln s_{\min} - \{\ln(1-w\lambda) - \ln \rho_{\mathrm{G}} - \ln \sqrt{\eta^2+1}\}}{\sqrt{\ln(\eta^2+1)}} \leq z_{\theta_c}. \tag{9}$$

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Thus, the maximum building density that satisfies Equation (8) is derived as follows:

$$\rho_{\mathrm{G}} \leq \frac{1-w\lambda}{\sqrt{\eta^2+1}} \cdot \frac{\exp[z_{\theta_c}\sqrt{\ln(\eta^2+1)}]}{s_{\min}}$$

$$\equiv \rho_{\mathrm{G}}(\lambda; s_{\min}, \theta_c), \quad \text{for } \lambda \in (0, 1/w). \tag{10}$$

According to dimension analysis, the inverse of s_{mim} is equivalent to the building density based on the deterministic approach. However, the sup ρG based on the stochastic approach is not equal to $1/ s_{mim}$ because sup ρG is contingent not only on $1/ s_{mim}$ but also on λ, w, η and θ.

Moreover, by applying the same method of deriving Equation (9) to Equations (5) to (7), the ratio that a building lot frontage is below the minimum building lot frontage F_{mim} is less than ψc satisfies the following equation:

$$H(F \leq F_{\min}) = \int_0^{F_{\min}} h(F)dF$$

$$= \Phi\left(z_{\min} = \frac{\ln F_{\min} - \left\{\ln \lambda(2-w\lambda) - \ln \alpha\rho_{\mathrm{G}} - \ln\sqrt{\eta_F^2+1}\right\}}{\sqrt{\ln(\eta_F^2+1)}}\right)\vdots \tag{11}$$

where z_{min} denotes the standardisation of ln F_{mim} and ψc denotes the allowance for a narrower building lot frontage than F_{mim}. Thus, sup ρG that satisfies Equation (11) is derived as follows:

$$\rho_{\mathrm{G}} \leq \frac{\lambda(2-w\lambda)}{\alpha\sqrt{\eta_{\mathrm{F}}{}^2+1}} \cdot \frac{\exp[z_{\psi_c}\sqrt{\ln(\eta_{\mathrm{F}}{}^2+1)}]}{F_{\min}}$$

$$\equiv \rho_{\mathrm{G}}(\lambda; F_{\min}, \psi_c), \quad \text{for } \lambda \in (0, 2/w), \tag{12}$$

where $z\psi c$ the standardisation of ln F such that $\Phi(z\psi c) = \psi c$ respectively. In Equation (12), $\lambda(2-w\lambda)$ is proportional to the total length of urban block perimeters in a district. Given that $\lambda(2-w\lambda)$ is a convex upward quadratic function of λ, this is maximised if $\lambda = 1/w$. If the domain of λ ranges from 0 to $1/w$, then $\lambda(2-w\lambda)$ is the increasing function of λ. In actual districts in the Tokyo metropolitan region, λ is less than $1/w$. Therefore, $\rho_G(\lambda; F_{mim}, \psi c)$ is also the increasing function of λ. On the other hand, $\rho_G(\lambda; s_{mim}, \theta c)$ is the decreasing function of λ if the domain of λ ranges from 0 to $1/w$. Therefore, $\rho_G(\lambda; s_{mim}, \theta c)$ and $\rho_G(\lambda; F_{mim}, \psi c)$ are traded off in terms of λ (0, $1/w$).

Figure 1 presents the relationship between $\rho_G(\lambda; s_{mim}, \theta c)$ and $\rho_G(\lambda; F_{mim}, \psi c)$ with respect to θc and ψc. The values of the other parameters η, η_F and α are respectively 1.0, 0.6 and 1.2. The value of w ranges from (a) 6 metres, (b) 8 metres to (c) 10 metres. The values of s_{mim} and F_{mim} are respectively 100 m^2 and 6 m. It was found that for any θc and ψc, (1) the relationship between $\rho_G(\lambda; s_{mim}, \theta c)$ and $\rho_G(\lambda; F_{mim}, \psi c)$ shows a trade-off in terms of λ; (2) for any λ, the greater the value of w, the smaller the values of $\rho_G(\lambda; s_{mim}, \theta c)$ and $\rho G(\lambda; F_{mim}, \psi c)$; (3) for any λ, the greater the value of η, the smaller the value of $\rho G(\lambda; s_{mim}, \theta)$; and (4) for any λ, the greater the values of ηF and α, the smaller the value of $\rho G(\lambda; F_{mim}, \psi c)$, respectively. Hence, for any pair of θc and ψc, we can obtain the sup ρG and optimal λ^* as the point of intersection of $\rho G(\lambda; s_{mim}, \theta c)$ and $\rho G(\lambda; F_{mim}, \psi c)$. As mentioned above, the values of η, ηF and α are stable. Furthermore, the change in the value of w does not affect the shape of $\rho G(\lambda; s_{mim}, \theta c)$ and $\rho G(\lambda; F_{mim}, \psi c)$. Therefore, in Equations (10) and (12), the values of η, ηF, w and α are regarded as constant irrespective of districts.

In addition, as shown in Figure 2 (drawn in grey domain), $\rho G \le \rho G(\lambda; F_{mim}, \psi c)$ $(\lambda \le \lambda^*)$ and $\rho G \le \rho G(\lambda; s_{mim}, \theta c)$ $(\lambda^* \le \lambda)$ is a feasible set of the pair of ρG and λ that satisfies Equations (10) and (12). Moreover, the black circle represents the coordinate of the sup ρG and optimal λ^* as the point of intersection of $\rho G(\lambda; s_{mim} = 100, \theta c = 0.4)$ and $\rho G(\lambda; F_{mim} = 6, \psi c = 0.3)$. This feasible set can be regarded as a set of appropriate pairs of ρG and λ because both $\theta < \theta c$ and $\psi < \psi c$ are satisfied. The criteria of s_{mim} and F_{mim} should be determined from the perspective of a residential and built environment's levels of sanitation and safety at a building lot scale. In addition, not only should a civil minimum criterion be considered in order to improve the residential environment of a district, but also a higher criterion should be used to increase its desirability. On the other hand, the values of θc and ψc can be given as a temporary desirable level (e.g., based on policy making). For instance, consider a district in which θ and ψ are 0.6 and 0.5 (shown as a black triangle in Figure 2), respectively. As the first step level, the values of θc and ψc are respectively set as 0.4 and 0.3. Furthermore, as the second step level, the values of θc and ψc are respectively set as 0.2 and 0.1 (shown as a white circle in Figure 2). This step-by-step approach is expected to provide urban planners with a theoretical basis to discuss the validity of present and future density values in terms of improving the residential environment.

3. Discussion

In this section, a framework for discussing the rationale behind current building and road network densities will be proposed on the basis of Figure 2, before applying it to the empirical study of districts in the Tokyo metropolitan region. As shown in Figure 2, the first quadrant can be partitioned into four sub-regions called regions (A), (B), (C) and (D). Figure 3 illustrates each approach to reducing θ and ψ as arrows on sub-regions (B), (C) and (D), respectively. The start and end points of each arrow represent the present and improved pair of the present building density and road network density, respectively.

In region (B), although Equation (12) is satisfied, Equation (10) is not satisfied. In other words, even though the ratio of frontages shorter than F_{mim} is below ψc, the ratio of building lot sizes smaller than s_{mim} is not below θc. Thus, if a district is categorised into region (B), the policy for decreasing building density in order for this district to be categorised into region (A) should be adopted. To reduce θ from the present $\theta > \theta c$ to θc, two approaches are available: (B1) decreasing ρG and fixed λ parallel to the vertical axis to the region (A); or (B2) decreasing ρG and λ along the curve of $\rho G(\lambda; F_{mim} = 6, \psi < \psi c)$ to the region (A). On the one hand, by adopt-

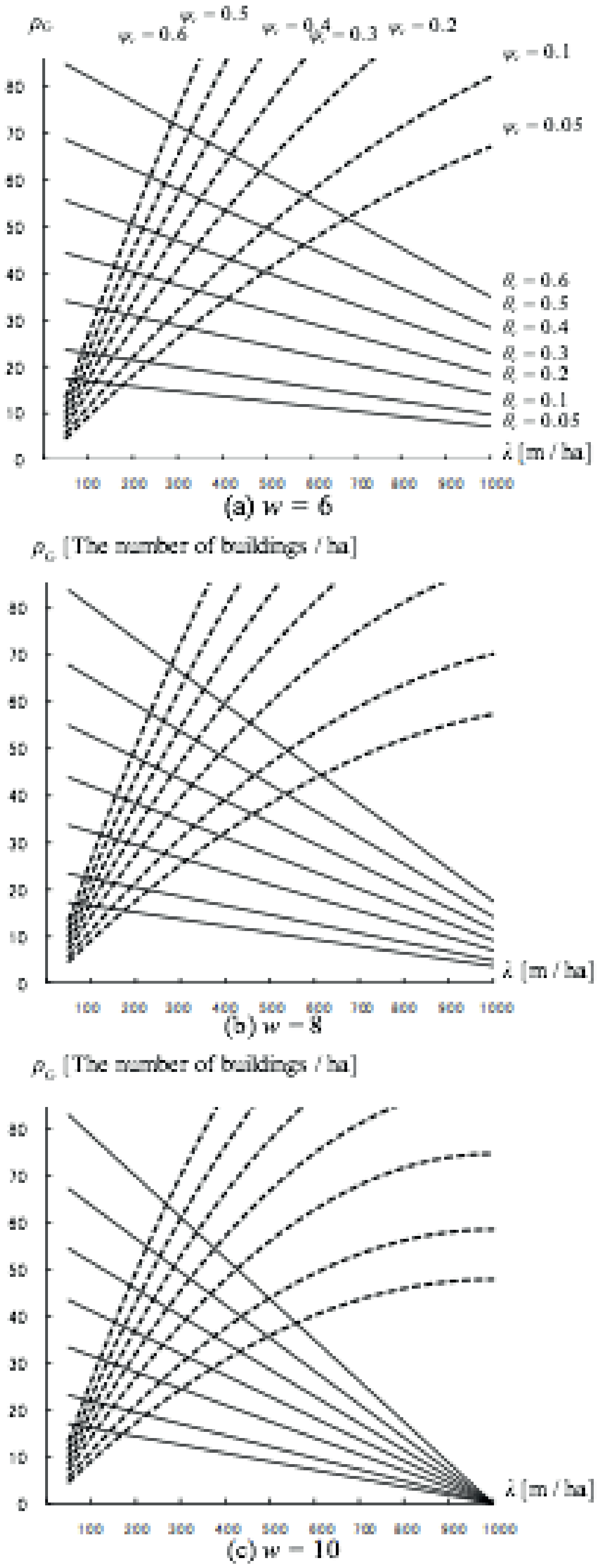

Figure 1. *Relationships between the sup* ρG *and* λ*, given as Equations (10) (solid lines) and (12) (dotted lines) with respect to* θc *and* ψc*. The values of other parameters,*η*,* ηF *and* α*, are respectively 1.0, 0.6 and 1.2, while* $s_{mim} = 100$ *m² and* $F_{mim} = 6$ *metres. The value of w ranges from (a) 6 metres, (b) 8 metres to (c) 10 metres.*

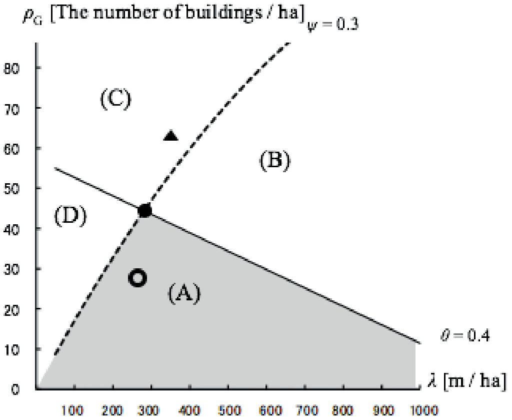

Figure 2. *A feasible set of the pair of ρG and λ (drawn in grey domain). The solid line and dotted line represent the sup ρG given as Equations (10) and (12), respectively. The values of other parameters (η, ηF, α and w) are respectively 1.0, 0.6, 1.2 and 8 metres while smim = 100 m2 and Fmim = 6 metres. The values of θc and ψc are 0.4 and 0.3, respectively. The coordinate of the sup ρG and optimal λ* as the point of intersection of ρG(λ; smim = 100, θc = 0.4) and ρG(λ; Fmim = 6, ψc = 0.3) as the black circle.*

ing approach (B2), the ratio of building lots smaller than s_{mim} can be reduced by decreasing both ρG and λ simultaneously, with ψ (< ψc) being held constant. Decreasing λ is equivalent to increasing at the expense of the road area of a district. Thus, approach (B2) corresponds to a land readjustment project that merges small urban blocks into a large urban block, as well as improving the pattern and width of road networks in order to meet demand for large building lots. On the other hand, by adopting approach (B1), θ and ψ can be reduced without decreasing λ by merging several adjacent building lots that are narrower than F_{mim} – especially *flag-shaped building lots* – into a regular building lot. Thus, approach (B1) corresponds to land readjustment projects of an urban block without altering road network patterns.

In region (D), although Equation (10) is satisfied, Equation (12) is not satisfied. Thus, if a district is categorised into region (D), the policy to either reduce building density or increase road network density so that it may be categorised into region (A) should be adopted. To decrease from the present ψ > ψc to ψc, two approaches exist: (D1) decreasing ρG and fixed λ parallel to the vertical axis to the region (A); or (D2) decreasing ρG but increasing λ along the curve of ρG(λ; s_{mim} = 100, θ < θc) to the region (A). Whereas approach (D2) renders θ constant, approach (D1) brings θ close to 0. In terms of improving the residential environment, approach (D1) is more appropriate because it is possible to reduce both small and narrow building lot frontages. Furthermore, owing to rigid budget constraints for road construction and maintenance, approach (D1) is more practical than approach (D2).

In region (C), both Equations (10) and (12) are not satisfied. Thus, if a district is categorised into region (C), the policy to reduce building density in order that it may be categorised into region (A) should be adopted. To decrease from the present θ ≥ θc to θc and from the present ψ ≥ ψc to ψc, two approaches exist: (C1) decreasing ρG and fixed λ parallel to the vertical axis to the region (A); (C2) decreasing ρG and λ to the maximum ρG and λ*, respectively; or (C3) decreasing ρG but increasing λ to the maximum ρG and λ*, respectively. Whereas approaches (C2) and (C3) accompany either increasing or decreasing the present λ to λ*, approach (C1) does not. In terms of decreasing the present ρG effectively, approaches (C2) and (C3) are more appropriate than approach (C1). This is because by adopting approach (C3), although additional roads are required, the reduction of ρG in approach (C3) is smaller than that in approach (C1).

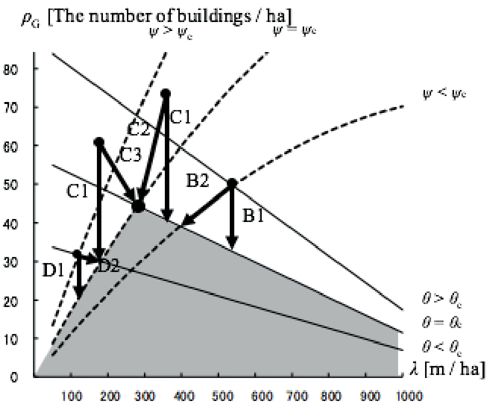

Figure 3. *Approaches to reducing small and narrow building lots in each sub-region.*

In order to discuss the rationale behind the present values of ρG and λ in districts in the Tokyo metropolitan region (the total number of which is 3,117), the figures are plotted as shown in Figure 4 (black dots). It was found that (1) if θc and ψc are 0.4 and 0.3 (a desirable level), respectively, almost all of the districts in the Tokyo metropolitan region are categorised into either region (A) or (B); and (2) if θc and ψc are 0.1 and 0.1 (the more desirable level), respectively, almost all of the districts in the Tokyo metropolitan region are categorised into regions (A), (B) or (C).

As mentioned in the previous section, if the combination of ρG and λ in a district is categorised into region A, it can be regarded as appropriate because both θ < θc and ψ < ψc are satisfied. Otherwise, the policy for either decreasing ρG or increasing λ in order for this district to be categorised into the region (A) should be adopted.

4. Conclusion

The substantive motivation of this research was to discuss the rationale behind current building and road network densities by considering variation in building lot sizes and frontages based on a stochastic approach. The criteria of s_{mim} and F_{mim} were deemed desirable, and θc and ψc were set as a temporary desirable level (e.g., based on policy making), enabling us to optimise building density and road network density at a district scale.

Optimal building density and road network density can be obtained as the point of intersection of ρG(λ; F_{mim}, ψc) and ρG(λ; s_{mim}, θc). Furthermore, ρG ≤ ρG(λ; F_{mim}, ψc) (λ ≤ λ*) and ρG ≤ ρG(λ; s_{mim}, θc) (λ* ≤ λ) can be delineated as a set of appropriate ρG and λ because both θ < θc and ψ < ψc are satisfied. Moreover, in order to discuss the rationale behind the present values of ρG and λ in districts in the Tokyo metropolitan region, they are categorised into the four regions (A) (appropriate), (B) (θ > θc and ψ < ψc), (C) (θ > θc and ψ > ψc) and (D) (θ < θc and ψ > ψc). It was found that: (1) if θc and ψc are 0.4 and 0.3 (a desirable level), respectively, almost all of the districts in the Tokyo metropolitan region are categorised into either region (A) or (B); and (2) if θc and ψc are 0.1 and 0.1 (the more desirable level), respectively, almost all of the districts in the Tokyo metropolitan region are categorised into either region (A), (B) or (C). As mentioned in the previous section, if the combination of ρG and λ in a district is categorised into region A, it can be regarded as appropriate because both θ < θc and ψ < ψc are satisfied.

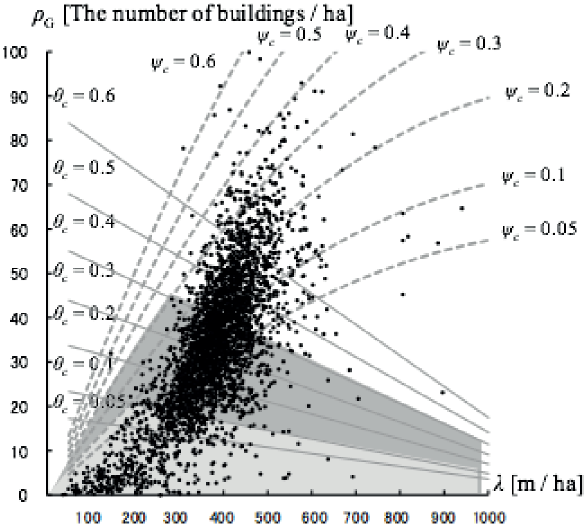

Figure 4. *Building density and road network density of districts in the Tokyo metropolitan region (black dots) with $\rho G(\lambda$; smim = 100, θc) and $\rho G(\lambda$; Fmim = 6, ψc) (drawn in solid lines and dotted lines, respectively). The values of other parametersη, ηF, α and w) are respectively 1.0, 0.6, 1.2 and 8 metres, while smim = 100 m2 and Fmim = 6 metres. The number of districts is 3,117. The regions drawn in dark grey and light grey are a feasible set of the pair of ρG and λ for θc = 0.4 and ψc = 0.3 and for θc = 0.1 and ψc = 0.1, respectively.*

Otherwise, the policy for either decreasing building density or increasing road network density in order for this district to be categorised into the region (A) should be adopted.

As mentioned in the second section, the inverse of s_{mim} is equivalent to building density based on the deterministic approach. However, the sup ρG based on the stochastic approach is not equal to 1/ s_{mim} because sup ρG relies not only on 1/ s_{mim} but also on λ, *w*, η and θ. According to Figure 4, although building density is sixty buildings per hectare, various pairs of θc and ψc can be observed – e.g., (θc, ψc) = (0.5, 0.6), (0.6, 0.3) – implying that building density does not capture the ratio of smaller and narrower building lots on its own. By introducing road network density, we can understand the variation in urban form and the ratio of smaller and narrower building lots below the given criterion of s_{mim} and F_{mim}. This is a limitation of solely using building density because it does not allow us to understand the variation in urban form. On the other hand, in the districts of the Tokyo metropolitan region and given the values of θc and ψc as 0.6 and 0.3, respectively, a building density below sixty buildings per hectare enables districts in the Tokyo metropolitan region to be categorised into region (A) (appropriate).

Several points can be suggested for future research. First, variation in other dimensions such as building lot depths and building height also help characterise variability in urban form. By considering the ratio of deeper building lot depths than the maximum criterion being less than permitted, region (A) might be more restricted than when this ratio is not used. Such future work should thus explore the potential of *Spacematrix*. Second, based on the characterisation of variations in urban form on the sub-space of *Spacematrix*, the rationale for contemporary zoning regulations regarding each building lot (e.g. BCR, FAR and building setbacks) could be reconsidered to promote a better residential environment.

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